Multiple Integral: It is defined as a definite integral of a function of more than one real variables. Integrals of a function of two variables over a region in R^2 are called double integrals. $I = \int_a^b\int_c^d f(x,y)\,dx\,dy$

Integrals of a function of three variables over a region of R^3 are called triple integrals. $I = \int_a^b\int_c^d\int_e^f f(x,y,z)\,dx\,dy\,dz$

Multiple Integral.

- Double integral or Surface integral (Area)
- Triple Integral. or Volume Integral. (volume).

① $I = \int_a^b f(x)\,dx$ let $f(x) = \phi'(x)$.

$\Rightarrow \int_a^b \phi'(x)\,dx = \left[\phi(x)\right]_a^b = \phi(b) - \phi(a).$

② $I = \underbrace{\int_a^b \underbrace{\int_c^d f(x,y)\,dx}_{\text{limits of } x}\,dy}_{\text{Limits of } y}$ or $\int_{a \to y}^{b}\int_{c \to x}^{d} f(x,y)\,dx\,dy.$

$I = \int_a^b\int_c^d f(x,y)\,dy\,dx$ or $\int_{a \to x}^{b}\int_{c \to y}^{d} f(x,y)\,dy\,dx$

③ $I = \int_0^1 \underbrace{\int_{x^2}^{x} f(x,y)\,dx}_{y = f(x)}\,dy \Rightarrow \underbrace{\int_0^1 \underbrace{\int_{x^2}^{x} f(x,y)\,dy}_{y}\,dx}_{x}$

limits of y.

$f(x,y,z) = 3D$ Personal page (2)

(4) $I = \int_0^1 \int_x^{x^2} \int_{x-y}^{x+y} f(x,y,z)\, dx\, dy\, dz$.

$x \downarrow f(x) \downarrow y = f(x) \downarrow$ limits of y.

$x-y \downarrow f(x,y) \downarrow z = f(x,y) \downarrow$ limits of 'z'

$$\Rightarrow \int_0^1 \int_x^{x^2} \int_{x-y}^{x+y} f(x,y,z)\, dz\, dy\, dx$$

z

y

x

(5) If $I = \int_0^1 \int_0^1 \int_0^1 dx\, dy\, dz$ Then how many ways to solve this integral. (Change of order)

$$I = \int_0^2 \int_0^{2-x} \int_0^{4-2x-2y} 2x\, dx\, dy\, dz = ?$$

Sta...

1. $\displaystyle\int_0^1 \int_1^2 dx\, dy$

2. $\displaystyle\int_1^2 \int_0^3 (x+y)\, dx\, dy$

3. $\displaystyle\int_2^4 \int_1^2 (x^2+y^2)\, dy\, dx$

4. $\displaystyle\int_0^1 \int_{x^2}^{x} xy^2\, dy\, dx$

5. $\displaystyle\int_1^2 \int_0^{y^{3/2}} \frac{x}{y^2}\, dx\, dy$

6. $\displaystyle\int_0^1 \int_x^{\sqrt{x}} (y+y^3)\, dy\, dx$

7. $\displaystyle\int_0^1 \int_0^{x^2} x e^y\, dy\, dx$

8. $\displaystyle\int_2^4 \int_y^{8-y} y\, dx\, dy$

9. $\displaystyle\int_0^4 \int_{y/2}^2 e^{x^2}\, dx\, dy$

10. $\displaystyle\int_0^2 \int_{y^2}^4 y \cos x^2\, dx\, dy$